hep-th/9910059

The antisymmetric tensor propagator in AdS

Iosif Bena
University of California, Santa Barbara, CA 93106
iosif@physics.ucsb.edu

(October 7, 1999)

Abstract

In this brief note we construct the propagator for the antisymmetric tensor in AdS_{d+1}. We check our result using the Poincaré duality between the antisymmetric tensor and the gauge boson in AdS_5. This propagator was needed for a computation which turned out to be too hard. It can be used for computing various other things in AdS.

Typeset using REVTEX

I. INTRODUCTION

In [1,2], a lot of effort was put into finding the AdS propagators for the graviton and the gauge boson. Their methods can be used straightforwardly for the $B_{\mu\nu}$ propagators. An ansatz can be made for bitensor propagators. This ansatz contains both gauge artifacts and gauge invariant parts. Upon using the equation of motion for $B_{\mu\nu}$ we obtain an equation for the gauge invariant part of the propagator, whose solution is hypergeometric. For d=5 it simplifies to an algebraic function of the chordal distance. As explained in [1], working on the subspace of conserved sources makes gauge fixing unnecessary. We check our result by verifying the 5-dimensional Poincaré duality between A_μ and $B_{\mu\nu}$.

II. THE $B_{\mu\nu}$ PROPAGATOR

In Euclidean AdS_{d+1}, with the metric

$$ds^2 = \frac{1}{z_0^2}(dz_0^2 + \Sigma_{i=1}^d dz_i^2), \tag{1}$$

the easiest way to express invariant functions and tensors is in terms of the chordal distance:

$$u \equiv \frac{(z_0 - w_0)^2 + (z_i - w_i)^2}{2z_0 w_0}. \tag{2}$$

The action for an antisymmetric 2-tensor coupled to a conserved source $S_{\mu\nu}$ is:

$$S_B = \int d^{d+1}z\sqrt{g}[\frac{1}{2\cdot 3!}H^{\mu\nu\rho}H_{\mu\nu\rho} - \frac{1}{2}B_{\mu\nu}S^{\mu\nu}], \tag{3}$$

where

$$H_{\mu\nu\rho} = D_\mu B_{\nu\rho} + D_\nu B_{\rho\mu} + D_\rho B_{\mu\nu}. \tag{4}$$

The Euler Lagrange equation has a solution of the form:

$$B_{\mu\nu}(z) = \frac{1}{2}\int d^{d+1}w\sqrt{g}G_{\mu\nu;\mu'\nu'}(z,w)S^{\mu'\nu'}(w), \tag{5}$$

where $G_{\mu\nu;\mu'\nu'}$ is the bitensor propagator. To simplify notation, the D's with unprimed indices mean covariant derivatives with respect to z, and those with primed indices with respect to w. The equation $G_{\mu\nu;\mu'\nu'}$ satisfies is:

$$\begin{aligned} D^\rho(D_\mu G_{\nu\rho;\mu'\nu'} + D_\nu G_{\rho\mu;\mu'\nu'} + D_\rho G_{\mu\nu;\mu'\nu'}) &= -\delta(z,w)(g_{\mu\mu'}g_{\nu\nu'} - g_{\mu\nu'}g_{\nu\mu'}) + \\ &+ D_{\mu'}\Lambda_{\mu\nu;\nu'} - D_{\nu'}\Lambda_{\mu\nu;\mu'}, \end{aligned} \tag{6}$$

where $\Lambda_{\mu\nu;\nu'}$ is a diffeomorphism whose contribution vanishes when integrated against the covariantly conserved source $S^{\mu\nu}$. We can see that all of our bitensors are antisymmetric at both points.

Similarly to the methods in [1] we observe that a suitable basis for antisymmetric bitensors is given by:

$$T^1_{\mu\nu;\mu'\nu'} = \partial_\mu\partial_{\mu'}u\partial_\nu\partial_{\nu'}u - \partial_\mu\partial_{\nu'}u\partial_\nu\partial_{\mu'}u \tag{7}$$

$$T^2_{\mu\nu;\mu'\nu'} = \partial_\mu\partial_{\mu'}u\partial_\nu u\partial_{\nu'}u - \partial_\mu\partial_{\nu'}u\partial_\nu u\partial_{\mu'}u - \partial_\nu\partial_{\mu'}u\partial_\mu u\partial_{\nu'}u + \partial_\nu\partial_{\nu'}u\partial_\mu u\partial_{\mu'}u.$$

Thus, an ansatz for G is $G = T^1F^1(u) + T^2F^2(u)$. Nonetheless, we use a different decomposition, which illustrates better the gauge artifacts

$$G_{\mu\nu;\mu'\nu'} = T^1_{\mu\nu;\mu'\nu'}H(u) + D_\mu V_{\nu;\mu'\nu'} - D_\nu V_{\mu;\mu'\nu'}, \tag{8}$$

where $V_{\mu;\mu'\nu'} = Y(u)[\partial_\mu\partial_{\mu'}u\partial_{\nu'}u - \partial_\mu\partial_{\nu'}u\partial_{\mu'}u]$. Also, an antisymmetric $\Lambda_{\mu\nu;\nu'}$ can be expressed as

$$\Lambda_{\mu\nu;\nu'} = A(u)[\partial_\nu\partial_{\nu'}u\partial_\mu u - \partial_\mu\partial_{\nu'}u\partial_\nu u]. \tag{10}$$

We can now substitute (8) and (10) in (6), and after a long computation we obtain

$$D^\rho(D_\mu G_{\nu\rho;\mu'\nu'} + D_\nu G_{\rho\mu;\mu'\nu'} + D_\rho G_{\mu\nu;\mu'\nu'}) - D_{\mu'}\Lambda_{\mu\nu;\nu'} + D_{\nu'}\Lambda_{\mu\nu;\mu'} =$$

$$= T^1[H''u(u+2) + H'(1+u)(d-1) - 2A] - T^2[H''(1+u) + H'(d-1) + A']. \tag{11}$$

For $z \neq w$, we obtain 2 equations by setting the scalar coefficients of the two tensors to 0. We can observe that the $V_{\mu;\mu'\nu'}$ part which was a gauge artifact dropped out as expected. Thus, for $u \neq 0$ we have the equations:

$$H''u(u+2) + H'(1+u)(d-1) - 2A = 0 \tag{12a}$$

$$H''(1+u) + H'(d-1) + A' = 0. \tag{12b}$$

The second equation can be integrated once, with the integration constant chosen so that A and H vanish as $u \to \infty$. Combining this with (12a) we find the differential equation obeyed by H:

$$u(2+u)H''(u) + (d+1)(u+1)H'(u) + 2(d-2)H = 0. \tag{13}$$

This equation is hypergeometric, but for $d = 4$ the solution which vanishes as $u \to \infty$ is

$$H(u) = \frac{\Gamma((d-1)/2)}{4\pi^{(d+1)/2}} \frac{u+1}{[u(u+2)]^{(d-1)/2}}, \tag{14}$$

properly normalized to take care of the δ function in (6).

III. POINCARÉ DUALITY

In 5 dimensions a 2-form is Poincaré dual with a gauge boson, by the relation:

$$H_{\mu\nu\rho}\epsilon^{\mu\nu\rho\sigma\lambda} = 3!\ F^{\sigma\lambda} \tag{15}$$

Therefore, we expect:

$$\langle F^{\sigma\lambda}(z)F^{\sigma'\lambda'}(w)\rangle = \frac{1}{(3!)^2}\epsilon^{\mu\nu\rho\sigma\lambda}\epsilon^{\mu'\nu'\rho'\sigma'\lambda'}\langle H_{\mu\nu\rho}(z)H_{\mu'\nu'\rho'}(w)\rangle. \tag{16}$$

Checking (16) is a verification that our result is true. We use the fact that

$$\langle B_{\mu\nu}B_{\mu'\nu'}\rangle = G_{\mu\nu;\mu'\nu'} \tag{17}$$

and

$$\langle A_\mu A_{\mu'} \rangle = G_{\mu;\mu'}, \tag{18}$$

where the second propagator was found in [2]. We could check the tensor equality (16) term by term, but it is messy. We rather observe that the right hand side of (16) is a bitensor antisymmetric at both ends, and thus it will have the structure

$$\epsilon^{\mu\nu\rho\sigma\lambda}\epsilon^{\mu'\nu'\rho'\sigma'\lambda'}\langle H_{\mu\nu\rho}(z)H_{\mu'\nu'\rho'}(w)\rangle = F_1(u)T_1^{\mu\nu;\mu'\nu'} + F_2(u)T_2^{\mu\nu;\mu'\nu'}. \tag{19}$$

Concentrating on the components of $\langle F^{z_0 z_i}(z)F^{z_0' z_j'}(w)\rangle$ we obtain:

$$2F_1 + F_2(1+u) = H'', \tag{20a}$$

$$F_2(1+u)^2 + F_2 + 2F_1(1+u) = 2H''(1+u) + 3H', \tag{20b}$$

which give the same F_1 and F_2 as the ones obtained from the gauge propagator derived in [2].

IV. CONCLUSION

We computed the propagator for $B_{\mu\nu}$ in AdS_{d+1} and checked our result by using Poincaré duality for $d = 4$. This propagator can be used for computing various quantities having to do with $B_{\mu\nu}$ charged objects (like strings or D-branes with electric flux) in AdS.

Acknowledgements: I'd like to acknowledge useful conversations with Joe Polchinski, Gary Horowitz and Veronika Hubeny. This work was supported in part by NSF grant PHY97-22022.

APPENDIX A: SEVERAL USEFUL IDENTITIES INVOLVING THE CHORDAL DISTANCE

In the computations the following identities were useful:

$$\partial_\mu \partial_{\nu'} u = \frac{-1}{z_0 w_0}[\delta_{\mu\nu'} + \frac{(z-w)_\mu \delta_{\nu' 0}}{w_0} + \frac{(w-z)'_\nu \delta_{\mu 0}}{z_0} - u\delta_{\mu 0}\delta_{\nu' 0}] \tag{$A1$}$$

$$\partial_\mu u = \frac{1}{z_0}[(z-w)_\mu / w_0 - u\delta_{\mu 0}] \tag{$A2$}$$

$$\partial_{\nu'} u = \frac{1}{w_0}[(w-z)_{\nu'}/z_0 - u\delta_{\nu' 0}] \tag{$A3$}$$

$$D^\mu \partial_\mu u = (d+1)(u+1) \tag{$A4$}$$

$$\partial^\mu u \; \partial_\mu u = u(u+2) \tag{$A5$}$$

$$D_\mu \partial_\nu u = g_{\mu\nu}(u+1) \tag{$A6$}$$

$$(\partial^\mu u)(D_\mu \partial_\nu \partial_{\nu'} u) = \partial_\nu u \partial_{\nu'} u \tag{$A7$}$$

$$(\partial^\mu u)(\partial_\mu \partial_{\nu'})u = (u+1)\partial_{\nu'} u \tag{$A8$}$$

$$D_\mu \partial_\nu \partial_{\nu'} u = g_{\mu\nu}\partial_{\nu'} u \tag{$A9$}$$

REFERENCES

[1] Eric D'Hoker, Daniel Z. Freedman, Samir D. Mathur, Alec Matusis, Leonardo Rastelli; hep-th/9902042

[2] Eric D'Hoker, Daniel Z. Freedman; hep-th/9809179